A Recursive Scheme for MESFET Nonlinear Current Coefficient Evaluation Applied in Volterra-Series Analysis

Chien-Chang Huang and Han-Ting Pai

Department of Communication Engineering, Yuan Ze University
135, Yuan-Tung Rd., Chung-Li, Taoyuan 320, Taiwan, Republic of China

Abstract — **A recursive scheme for the two-dimensional Taylor-series coefficient evaluation of MESFET drain current characteristics is proposed by using low-frequency harmonic power measurements with the associated phase polarity information. The formulation is in a systematic approach to acquire the general expressions for various order terms. Numerical studies examine the validity of the proposed method with the additional measurement uncertainty simulations. Experimental verifications with the two-tone intermodulation distortion of a MESFET amplifier are also shown.**

I. Introduction

The Volterra-series has been widely studied and is proved as an efficient method for circuit analysis under weakly nolinearities [1]-[4]. The accurate modeling and associated extraction methods, however, are not so complete compared with the models used in harmonic-balance-based simulators. For example, the cross-effects of the v_{gs} and v_{ds} on the drain current are often neglected, and the degree of coefficients seldom larger than three. These simplifications may cause some errors especially for circuit intermodulation distortion prediction [5].

The drain current of a MESFET generally can be expressed in a two-dimensional Taylor series with the coefficients computed by the derivatives of *i-v* characteristics. This direct differentiating method may yields large error as higher order terms considered. The harmonic power measurement at low frequency is used in [6] to solve the problem and determine the MESFET nonlinear transconductances up to the third-order. Two-tone approach is proposed in [7] for measuring the v_{gs}/v_{ds} cross-effect of the drain current, and all the coefficients of the two-dimensional Taylor series can be obtained. Other methods to acquire the two-dimensional coefficients using a single tone measurement with different circuit configurations can be found as [8]-[9].

This paper presented a systematic extraction procedure in a recursive scheme for the two-dimensional nonlinear coefficients of MESFET drain current characteristics. It is based on the low frequency harmonic power measurement with the associated phase polarity information in different attenuation conditions for constructing the equation sets. The term "recursive" means that the extracted parameters are solved by using the former extracted data with some expressions, and the results become the known parameters for next order evaluation. This approach takes the advantage of reducing the number of unknowns in each solving process. With the systematic formulation of the extraction equation, the general forms of expressions for various order coefficients are acquired. The least-square method of the over-determined equation sets is utilized to optimize the best solution among the measured data. Some numerical techniques such as the pseudo-inverse method [10] can also be considered as the matrix in ill conditioned. The extraction procedure then are simulated to the fifth-order with some input data variations as measurement uncertainties to illustrate the error mechanism of the expanded coefficients.

For the experiment verification, a commercial MESFET is tested under certain biased condition. The two-dimensional fifth-order coefficients are evaluated based on the proposed method. In addition, the complete equivalent circuit model can be established for Volterra-series analysis with other linear components which values are given by the datasheet. To ensure the model's validity, a two-tone excitation of the MESFET is calculated by the Volterra-series analysis, and compared with the measured results. Good agreement between the simulations and measurements is achieved.

II. Extraction Method

Figure 1 shows the measurement equivalent circuit of the MESFET device with the low frequency input signal, where the output harmonics are monitored by a spectrum analyzer. The attenuator is designed to construct the equation sets for the parameter extraction via different value combinations of resistances. The relationships between the input/output harmonic powers and the nonlinear coefficients are derived as follows.

A. Nonlinear Current Expansions

0-7803-7695-1/03/$17.00 © 2003 IEEE

Under the weakly nonlinear assumption, the small signal drain current i_d of a MESFET can be represented by a two-dimensional Taylor series expansion at a certain biased point as

$$i_d(v_{gs}, v_{ds}) = \sum_{k=1}^{\infty} g_{mk} v_{gs}^k + \sum_{k=1}^{\infty}\sum_{\ell=1}^{\infty} g_{mkd\ell} v_{gs}^k v_{ds}^\ell + \sum_{\ell=1}^{\infty} + g_{dq} v_{ds}^\ell \, . \quad (1)$$

Meanwhile the control voltages v_{gs} and v_{ds} are expressed in series according to the Volterra theory [1]

$$v_{gs/ds} = \sum_{i=1}^{\infty} v_{gs/ds,i} \, , \quad (2)$$

where the subscript i is the orders of nonlinear effects. Substitute (2) into (1), the nonlinear current is decomposed to the summation of various nonlinear order terms, where the orders are determined by the voltage product terms in the forms of $(v_{gsi_1}^{k_1} v_{gsi_2}^{k_2} \cdots v_{gsi_n}^{k_n})(v_{dsj_1}^{l_1} v_{dsj_2}^{l_2} \cdots v_{dsj_m}^{l_m})$ with the value of $i_1k_1 + i_2k_2 + \cdots + i_nk_n + j_1l_1 + j_2l_2 + \cdots + j_ml_m$. Collect the same nonlinear order terms, the nonlinear currents are

$$\begin{aligned} i_{d2} &= g_{m2}v_{gs1}^2 + g_{d2}v_{ds1}^2 + g_{m1d1}v_{gs1}v_{ds1} \\ i_{d3} &= g_{m3}v_{gs1}^3 + 2g_{m2}v_{gs1}v_{gs2} + g_{d3}v_{ds1}^3 + 2g_{d2}v_{ds1}v_{ds2} \\ &\quad + g_{m1d1}(v_{gs1}v_{ds2} + v_{gs2}v_{ds1}) + g_{m2d1}v_{gs1}^2 v_{ds1} + g_{m1d2}v_{gs1}v_{ds1}^2 \\ &\cdots\cdots \end{aligned} \, . \quad (3)$$

Based on the current expressions in (3), one can develop the solving equations for the coefficients in a recursive manner by choosing the proper input powers and measuring the output harmonic powers as described in the following sub-sections.

B. Linear coefficient evaluation

Let the input signal in the sinusoidal form as

$$v_{in}(t) = V_{in}\cos(\omega t + \varphi) \, . \quad (4)$$

Similarly, v_{gs} and v_{ds} are now represented in phasor forms. The drain current then becomes

$$I_{d1} = g_{m1}V_{gs1} + g_{d1}V_{ds1} \, , \quad (5)$$

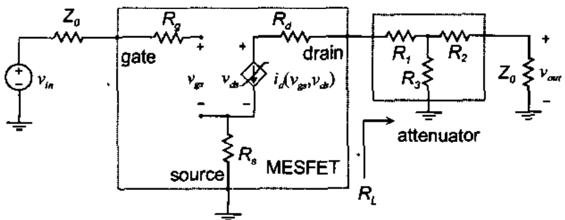

Fig. 1. Equivalent circuit for MESFET coefficient extraction measurement.

where the subscript 1 denotes the linear degree. By using the KVL and KCL in Fig. 1, the linear coefficient determining equation becomes

$$(V_{in} + \frac{R_s}{R_L'}V_{ds1})g_{m1} + V_{ds1}g_{d1} = -\frac{V_{ds1}}{R_L'} \, , \quad (6)$$

with the definition of $R_L' \equiv R_d + R_s + R_L$. Now the quantities g_{m1} and g_{d1} are regarded as unknowns, two equations can be established by choosing different attenuation resistances. The constants in (6) are obtained via the measurement from the spectrum analyzer and the available power of the signal source. The input powers should be tuned to proper values to ensure the linear condition. More than two measured data set in (6) can also be applied in the least-square sense for solving the linear coefficients.

C. Second and higher order coefficient evaluations

As the input power increase, the MESFET generates the second or higher harmonics from the nonlinear coefficients. The drain current in a N-order harmonic component is deduced from (3) in general forms as

$$\begin{aligned} I_{dN} &= I_{dNp} + I_{dNq} + I_{dNr} \\ I_{dNp} &= \frac{1}{2^{N-1}} \sum_{i=0}^{N} g_{m(N-i)di} V_{gs1}^{N-i} \cdot V_{ds1}^{i} \\ I_{dNr} &= g_{m1}V_{dsN} + g_{d1}V_{dsN} \end{aligned} \, . \quad (7)$$

The I_{dNq} is the only irregular expression that should be derived at certain nonlinear degrees. For convenience, they are listed in Table I up to the fifth-order terms. The N-order nonlinear coefficient determining equation then is

$$\sum_{i=0}^{N} (\frac{V_{gs1}}{V_{ds1}})^{N-i} g_{m(N-i)di} = \frac{-2^{N-1}[(1 + g_{m1}R_s + g_{d1}R_L')\frac{V_{dsN}}{R_L'} + I_{dNq}]}{V_{ds1}^N} \, , \quad (8)$$

with the nodal voltages V_{gsN} and V_{dsN} defined as

$$V_{dsN} = \frac{-I_{dNp}R_L'}{1 + g_{d1}R_L' + g_{m1}R_s} \, , \quad (9)$$

$$V_{gsN} = \frac{R_s}{R_L'}V_{dsN} \, . \quad (10)$$

The constant V_{dsN} that must be calculated from the measured harmonic power is given by

$$V_{dsN} = sign \cdot K\sqrt{2Z_0P_{out,N\omega}} \, , \quad (11)$$

TABLE I

I_{dNq} expressions up to the fifth-order terms

order N	I_{dNq} expression
2	0
3	$g_{m2}V_{gs1}V_{gs2}+g_{d2}V_{ds1}V_{ds2}+\frac{1}{2}g_{m1d1}(V_{gs1}V_{ds2}+V_{gs2}V_{ds1})$
4	$\frac{3}{4}(g_{m3}V_{gs1}^2V_{gs2}+g_{d3}V_{ds1}^2V_{ds2})+\frac{1}{2}[g_{m2}(V_{gs2}^2+2V_{gs1}V_{gs3})+g_{d2}(V_{ds2}^2+2V_{ds1}V_{ds3})]+$ $\frac{1}{4}[g_{m2d1}(V_{gs2}^2V_{ds2}+2V_{gs1}V_{gs2}V_{ds1})+g_{m1d2}(V_{ds2}^2V_{gs2}+2V_{ds1}V_{ds2}V_{gs1})]+\frac{1}{8}g_{m2d2}V_{gs1}^2V_{ds2}^2+\frac{1}{2}g_{m1d1}(V_{gs3}V_{ds1}+V_{gs2}V_{ds2}+V_{ds3}V_{gs1})$
5	$\frac{1}{2}(g_{m4}V_{gs1}^3V_{gs2}+g_{d4}V_{ds1}^3V_{ds2})+\frac{3}{4}[g_{m3}(V_{gs1}V_{gs2}^2+V_{gs1}^2V_{gs3})+g_{d3}(V_{ds1}V_{ds2}^2+V_{ds1}^2V_{ds3})]+g_{m2}(V_{gs2}V_{gs3}+V_{gs1}V_{gs4})+g_{d2}(V_{ds2}V_{ds3}+V_{ds1}V_{ds4})+$ $\frac{1}{8}[g_{m3d1}(V_{gs1}^3V_{ds2}+3V_{gs1}^2V_{gs2}V_{ds1})+g_{m1d3}(V_{ds2}^3V_{gs2}+3V_{ds1}^2V_{ds2}V_{gs1})]+\frac{1}{4}g_{m2d2}(V_{gs1}^2V_{ds1}V_{ds2}+V_{ds1}^2V_{gs1}V_{gs2})+$ $\frac{1}{4}g_{m2d1}(V_{gs1}^2V_{ds3}+2V_{gs1}V_{gs2}V_{ds2}+V_{gs2}^2V_{ds1}+2V_{gs1}V_{gs3}V_{ds1})+\frac{1}{4}g_{m1d2}(V_{ds1}^2V_{gs3}+2V_{ds1}V_{ds2}V_{gs2}+V_{ds2}^2V_{gs1}+2V_{ds1}V_{ds3}V_{gs1})+$ $\frac{1}{2}g_{m1d1}(V_{gs4}V_{ds1}+V_{gs3}V_{ds2}+V_{ds3}V_{gs2}+V_{ds4}V_{gs1})$

where K is the transformation factor from V_{dsN} to V_{out}. Note that the variable $sign=\pm 1$ represents the phase polarity of the N-harmonic output referenced to the input signal. It is due to the addition or subtraction operations in the drain current expression of (7) and can be decided through experiments of interference. The N degree coefficients then are solved based on (8) via various harmonic power measurements of different attenuations applied.

D. Numerical treatment for ill-condition matrix

The constants in the determining equation of (8) are in the form of $(V_{gs1}/V_{ds1})^n$, where the n is the nonlinear order number. It becomes quite small at high order coefficient extractions, and the matrix then is in ill conditioned. The pseudo-inverse method [10] is utilized to regularize the matrix by removing the too-small column vectors. The great improvement of the numerical stability then can be accomplished.

III. SIMULATION AND EXPERIMENT RESULTS

A. Simulation

Assume all the nonlinear coefficients are given with R_g, R_d, R_s values as the study case, the output harmonic powers can be computed via the nonlinear current expressions and associated output power relations. In addition, some measurement uncertainties can be simulated as the perturbations to the calculated power levels and the final results are treated as the input data to the recursive extraction procedure. The uncertainty influences of the harmonic power levels for the errors of extracted coefficients are plotted in Fig. 3 (a), (b), (c) and (d) to the fifth-order nonlinearities. One can find all in error-free without any power perturbations, and no greater than 50% deviations from the original values for all of the parameters even at 1.5 dB measured uncertainties in this study. The numerical results also show that the transconductances for various nonlinear orders are almost in the same deviation under measured uncertainties. The drain conductance g_d, however, behave quite different. Another interesting phenomenon is, for the cross-term coefficients g_{midj} with the same nonlinear degree, the coefficients with lager transconductance nonlinear index show the lower deviations in this case. This simulation examines the robustness of the proposed method and provides useful information for the error mechanism of the extraction parameters.

B. Experiment

The MESFET of SIEMENS CLY5 biased at $V_{gs}=-1.5$ V and V_{ds} = 3.0 V is used to demonstrated the extraction results. The nonlinear current coefficients are integrated with other linear components to establish the complete equivalent circuit model for Volterra-series analysis where the linear component values are given by the datasheet. A two-tone intermodulation distortion to the fifth-order nonlinearity then is simulated at about 1 GHz with the experimental comparison as shown in Table II. Good agreement between the calculated and measured data again validates the proposed method.

IV. CONCLUSION

This paper presented the extraction procedure of MESFET two-dimensional Taylor-series coefficients for drain currents with theoretical derivation, numerical simulation and experimental verification. The shown method is in a recursive scheme to reduce the number of unknowns in each solving process. The measured arrangements included the low-frequency harmonic power and the associated phase polarity in the advantages of easy implementations. The influences of the measurement uncertainties are studied via numerical simulations to examine the robustness of the proposed method. The final experiment verification is performed on the two-tone intermodulation distortion calculation up to the fifth-order terms, and good agreement with the measured data is achieved.

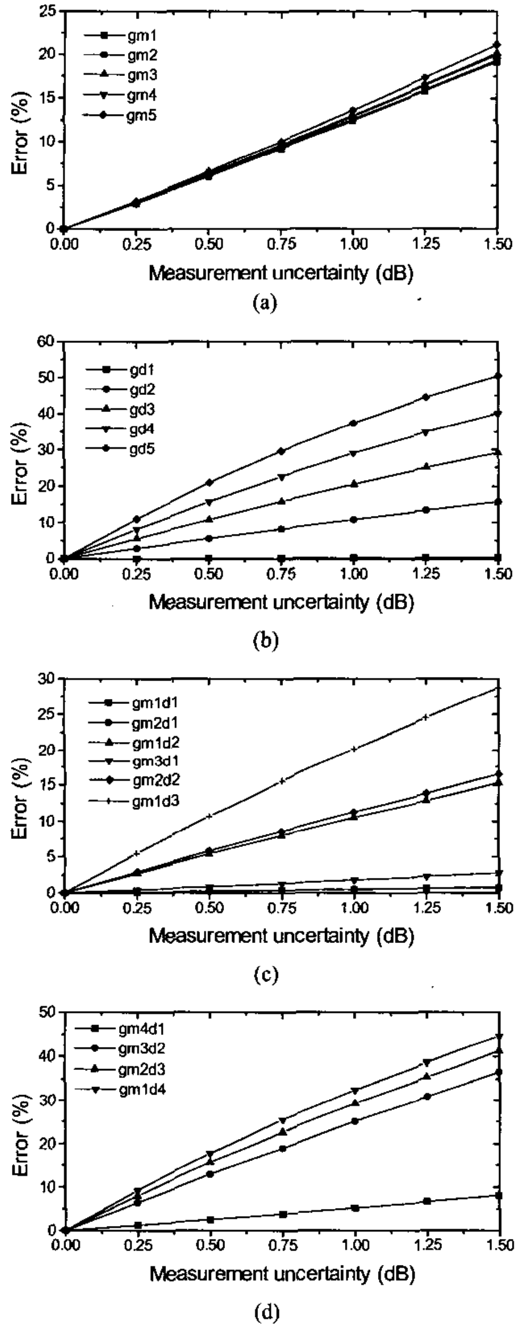

Fig. 2. Simulation results of extraction coefficient errors in power measurement uncertainties.

TABLE II

Calculated results for the SIEMENS CLY5 MESFET two-tone intermodulation using the extracted coefficients with measured data comparison, where f_1 = 1000 MHz, f_2 = 1001 MHz, input power = 1.35 dBm for each tone. The results are in unit of dBm.

	P_{f1}	P_{f2}	P_{2f1-f2}	P_{2f2-f1}	$P_{3f1-2f2}$	$P_{3f2-2f1}$
calculated	11.47	11.46	–21.99	–22.00	–49.58	–49.60
measured	12.06	11.94	–21.22	–21.58	–42.11	–42.86

REFERENCES

[1] J. J. Busogang, L. Ehrman and J. M. Graham, "Analysis of nonlinear systems with multiple inputs," *Proc. IEEE*, vol. 62, pp. 1088-1119, Aug. 1974.
[2] C. L. Law and C. S. Aitchison, "Prediction of wide-band power performance of MESFET distributed amplifiers using the Volterra series representation," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, pp. 1308-1317, Dec. 1986.
[3] C.-C. Huang and T.-H. Chu, "Analysis of MESFET injection-locked oscillators in fundamental mode of operation," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-42, pp. 1851~1857, Oct., 1994.
[4] S. T. Chew and T. Itoh, "Application of Volterra series to the problem of self-oscillating mixer," *IEEE Microwave Theory Tech.*, vol. MTT-44, pp. 269-274, Feb. 1996.
[5] A. Crosmun and S. A. Maas, "Minimization of intermodulation distortion in GaAs MESFET small-signal amplifiers," *IEEE Microwave Theory Tech.*, vol. MTT-37, pp. 1411-1417, Sept. 1989.
[6] S. A. Maas, "Modeling the gate I/V characteristics of a GaAs MESFET for Volterra-series analysis," *IEEE Microwave Theory Tech.*, vol. MTT-37, pp. 1134-1136, July 1989.
[7] J. C. Pedro and J. Perez, "Accurate simulation of GaAs MESFET's intermodulation distortion using a new drain-source current model," *IEEE Microwave Theory Tech.*, vol. MTT-42, pp. 25-33, Jan. 1994.
[8] G. Qu and A. E. Parker, "New model extraction for predicting distortion in HEMT and MESFET circuits," *IEEE Microwave & Guided Wave Letters*, vol. 9, pp. 363-365, Sept. 1999.
[9] C.-W. Fan and K.-K. M. Cheng, "A new method in characterizing the nonlinear current model of MESFETs using single-tone excitation," *IEEE MTT-S Int. Microwave Symp. Dig.*, pp. 449-452, 2000.
[10] M. M. Ney, "Method of moments as applied to electromagnetic problems," *IEEE Microwave Theory Tech.*, vol. MTT-33, pp. 972-980, Oct. 1985.